Let be a Graph

**Cuts**

A is a partition of into and such that (source) belongs to and (sink) belongs to.

The capacity of the cut is

A minimum cut is a cut whose capacity is minimum.

**Algorithm to find minimum cut**

Find maximum flow and define = {all vertices such that there exists a path from them to s in the final residual network} and . Then will be a minimum cut.

**Coverings, Matching, Independent Set**

Source: <https://www.epfl.ch/labs/dcg/wp-content/uploads/2018/10/GT-4-Covers.pdf>

**Preliminaries**

Bipartiteness:

A graph is bipartite if its vertices can be divided into two disjoints sets such that there is no edge between vertices of the same set.

**Necessary and sufficient condition:**

A graph is bipartite iff it doesn’t have and odd cycle.

**Definitions**

* **Matching :** Is a set such that the edges in M are pairwise disjoint
* **Vertex Cover:** Is a set such that every edge of is incident to a vertex of .
* **Edge Cover:** Is a set such that every vertex of is incident to an edge in (this concept is only defined in graph without isolated vertex)
* **Independent set:** Is a set such that no two vertices in are adjacent.

**Inequalities**

For any arbitrary Graph:

For any arbitrary Graph without isolated vertices:

**Gallai Theorem:**

For any arbitrary Graph:

For any arbitrary Graph without isolated vertices:

**Konig Theorem:**

Source: https://www.epfl.ch/labs/dcg/wp-content/uploads/2018/10/GT-3-Matchings.pdf

If the graph is bipartite,

If, additionally, doesn’t have isolated vertices,

**Algorithm for finding each of them in Bipartite Graph:**

Let say that our bipartite graph has the partition

* **Maximum matching**: Run the max flow algorithm on. All the edges between that have flow are edges of a maximum matching
* **Minimum edge cover**: Let denote the maximum matching size by |M|. Take the |M| edges of the maximum matching. For the other |V| - 2 |M| unmatched vertices, take one of its edges (the other endpoint must be matched). This set of edges is a minimum edge covering.
* **Minimum vertex cover**: Find a minimum cut . Take all the edges of the cut (those that goes from to ). All the vertices that belong to those edges (except from the source and the sink) form a minimum vertex cover.

(Source: http://theory.stanford.edu/~trevisan/cs261/lecture14.pdf)

* **Maximum Independent set:** Take all the vertices that are not in the minimum vertex cover. These vertices form a maximum independent set.